Buyer feedback as a filtering mechanism for reputable sellers

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We propose a continuum model for the description of buyer and seller dynamics in an Internet market. The relevant variables are the research effort of buyers and the sellers' reputation building process. We show that, if a commercial web-site gives consumers the possibility to rate credibly sellers they bargained with, vendors are forced to be more honest. This leads to mutual beneficial symbiosis between buyers and sellers; the overall enhanced volume of transactions contributes ultimately to the web-site, which facilitates the matchmaking service.

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1 The Problem

The Internet provides a new venue for commercial transactions, though there is still no consensus as to what fundamental mechanism makes a commercial web site tick or flop. In the law of supply and demand, transactions are beneficial to both buyers and sellers in general. However, so-called market failures can happen if the information about the quality of the product is very asymmetric.

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In the last few decades economists have made fundamental research work in this area —for instance the famous paper by George Akerlof on the "Lemon's Problem" [1] as well as more general applications of asymmetric information in many economic relationships by Joe Stiglitz et al.; for a recent review see [2].

What is special about Internet commerce? In the last few years the much initial enthusiasm turned into big disappointment, after many high flyers crashed and the Internet commerce bubble blowed. We feel that the fundamental mechanism is not yet generally appreciate. In this paper we want to highlight the unique role played by the reputation system. In the Internet commerce, the information asymmetry is extreme: buyers cannot evaluate the quality of products before purchasing them. Even worse, buyers don't even see sellers in their face, as in an off-line transaction. Thus the information asymmetry is much more severe than in the traditional commerce modes. The Internet, on the other hand, offers tremendous opportunities, since buyers can access a vast choice of products and the search costs are much reduced.

So, we face the dilemma: how to tap into the huge potential while avoiding the proverbial information asymmetry? Our analysis will show that the holy grail rests in binding the collective knowledge of all buyers about the sellers' reputation. The Internet commerce has the unprecedented potential to leverage the collective buying experience in a centralized place. Though a less than honest seller can get away with a questionable transaction on one buyer, the dissatisfied buyer can easily post his rating on this particular seller. Reputation is a valuable asset that no vendor can ignore. Indeed, most of the fast growing e-commerce web sites, including Internet auctions sites such as eBay [3], allow buyers to rate sellers after receiving the product they bid for. The ensemble of those ratings builds up a seller's reputation that can be viewed ever since by other buyers, thus replacing (and sometimes improving) the direct quality check of usual street shopping.

Our analysis is based on the fundamental conviction that sellers have the option of being honest or not. If the web site can establish a credible rating system to capture buyers' feedback, it effectively filters out dishonest sellers. There are no permanent cheaters, they must find other ways to make a living that are also beneficial to the society. The matchmaking service, provided by the web-site, facilitates a selection process that is the opposite of "adverse selection" [4]. Those sellers with good quality, would be encourage to join by a honest representation, while the dishonest ones wouldn't even want to try. Such a service can be easily rewarded, since such a web-site can take a slice from the mutually beneficial transactions. Our results show that the extent of the total transactions depends on the quality of the feedback rating system.

Our approach is to model buyers and sellers as two species in symbiosis, much like in population dynamics. In fact, our equations share many similarities with the well-known Lotka-Volterra model [5,6]. The key is to realize the two species have both converging as well as diverging interests: Buyers need sellers, the more the better, and vice versa. This much they have converging interests. But, in a particular transaction, a buyer's loss is the seller's gain. However, on the aggregate level transactions between the two groups are non-zero sum games —in fact positive sum games.

2 The Model

At time t, $B(t)$ potential buyers and $S(t)$ sellers meet in our virtual marketplace. Buyers' rationality is bounded by incomplete information and limited computing capability, but we retain the assumption of procedural rationality [7]. This means agents only dispose of a few options, but they are able to choose better ones with higher probability, provided they are given correct information. We shall consider a fast growing regime, assuming the number of buyers $B(t)$ grows exponentially in time. Regular users may decide to continue trading or stop doing so, according to their satisfaction at previous times.

The interest a given seller has in staying on that market can be easily estimated as a function of the earnings made. If he gained nothing, it is very probable that he won't repeat the experience. Sometimes he may lower his honesty in the hope to make more money, or increase it in order to sell more. On the other hand, buyers' profitability has to be inferred. Neglecting possible technical dysfunctions, delivery failures and other inconvenients not directly related to the actors of the transaction, there are two main possible sources of discontent: the item could differ from what the buyer was originally looking for, or its state of usage could be worse than he was promised. If a buyer is not satisfied, he is not likely to visit the web-site again in the near future.

Each seller s only sells products of a kind (x_s) and is characterized by his honesty h_s, which can be interpreted as the ratio quality/price he is selling at. His satisfaction $\gamma_s(t)$ is defined as the number of products sold $n_s(t)$, times the normalized unitary gain g_{h_s}:

$$\gamma_s(t) = n_s(t) g_{h_s} \tag{1}$$
$$g_{h_s} = 1 - h_s + m, \tag{2}$$

where m is the minimum profit margin, i.e. a percentage of the price that covers all expenses and leaves a revenue even to the most honest vendors. Here $1 - h_s$ can be regarded as an extra-profit that would equal zero in a perfectly competitive market. We assume no price discrimination on an individual basis, i.e. h_s does not depend on the particular buyer s is dealing with, even though this phenomenon may arise in particular contests [8].

Each buyer b looks, at time t, for a specifically desired product x_b. Products x_k, be they desired or sold, can be represented as elements of a metric space (real numbers or bit strings), where we can define a normalized distance $d_{b,s} = d(x_b, x_s) \in [0, 1)$ and an overlap $q_{b,s} = 1 - d_{b,s}$. The latter measures how close a product x_s is to the buyer's desire x_b. Now, if b buys a unit of $x_{\tilde{s}}$, once he receives it he is rewarded with a payoff $r_{b,\tilde{s}}$. His satisfaction $\gamma_b(t)$ then equals:

$$\gamma_b(t) = \begin{cases} r_{b,\tilde{s}} & \text{if } b \text{ purchased } x_{\tilde{s}} \text{ at time } t; \\ 0 & \text{if } b \text{ purchased nothing at time } t; \end{cases} \tag{3}$$

It is reasonable that $r_{b,s}$ be an increasing function of h_s (buyers are more satisfied if the purchased product has a better ratio quality/price) and $q_{b,s}$ (buyers are more satisfied if the purchased product is closer to their wishes). Hence we define

$$r_{b,s} = h_s q_{b,s}. \tag{4}$$

Finally, buyer b can rate seller's s honesty and influence his reputation. This happens for every buyer who deals with seller s at each time step, therefore one's reputation tends to his honesty h_s. Buyers, then, can take a look at sellers' reputation before purchasing a product. Whenever a difference between reputation and honesty is not explicitly mentioned, we shall assume they coincide. On the other hand, buyers are allowed to trust it or not, in a way we will describe later on.

Now we have a definition of buyers' and sellers' satisfaction. Their role becomes clear once we specify the dynamics: we will do that first, leaving the details of the transaction process for later sections. Since we are aiming to give a mean field description of the system, it is useful to introduce the average buyers' and sellers' satisfactions

$$\Gamma_B(t) = \sum_b \gamma_b(t)/B(t) \tag{5}$$

$$\Gamma_S(t) = \sum_s \gamma_s(t)/S(t). \tag{6}$$

When the number of buyers and sellers becomes large, the self-averaging effect yields $\sum_\zeta \gamma_\zeta(t) \simeq \sum_\zeta \bar{\gamma}_\zeta(t)$, with $\zeta = b, s$. Here the overline bar is an average over realizations: $\bar{\gamma}_\zeta(t)$ represents the average payoff an agent would get if he faced the same situation a great number of times. Let us assume h is a discrete variable which can only take H values, separated by a mesh $\Delta h = 1/(H-1)$. Then we can consider the number of sellers $S_h(t)$ belonging to a certain honesty class, with $S(t) = \sum_{h=0}^{1} S_h(t)$. Their average satisfaction will be

$$\Gamma_{S_h}(t) = \sum_{s:h_s=h} \gamma_s(t)/S_h(t), \text{ with:} \tag{7}$$

$$\Gamma_S(t) = \sum_h S_h(t)\Gamma_{S_h}(t)/S(t) = \langle \Gamma_{S_h}(t) \rangle. \tag{8}$$

Here and in the following, angular brackets stand for averages over the honesty distribution

$$p(h,t) = S_h(t)/S(t). \tag{9}$$

Notice that, while $\Gamma_B(t)$ is constrained in the range $[0,1]$, the value of Γ_{S_h} is only bounded by $(m+1)B(t)$. Now we are able to write $H+1$ replicator dynamics type [9] differential equations, describing the mean field time evolution of $B(t)$ and $S_h(t)$, for $h = 0, \Delta h, 2\Delta h, ..., 1$:

$$\frac{dB(t)}{dt} = c_B B(t) - [1 - \Gamma_B(t)]B(t) \tag{10}$$

$$\frac{dS_h(t)}{dt} = \Delta h[\Gamma_S(t) - 1]S(t) + [\Gamma_{S_h}(t) - 1]S_h(t). \tag{11}$$

Here the parameter c_B is a factor of growth, which embeds all the external conditions, such as liquidity and competition effects. Summing equation (11) over h we obtain

$$\frac{dS(t)}{dt} = 2[\Gamma_S(t) - 1]S(t). \tag{12}$$

The above equations arise from the following dynamics. At time t every buyer attracts c_B users in the web-site. Among the old clients a percentage $\Gamma_B \in [0,1]$ survives, while the others leave. In other words, the probability that an active buyer continues shopping in this market at future times is proportional to his satisfaction at time t. As for sellers, the term $\Delta h[\Gamma_S(t)-1]S(t)$ on the r.h.s. of equation (11) acts uniformly on every h level. Since $\Gamma_S(t)$ is the average profit a seller made at time t, it represents a general measure of profitability for the web-site. If it is bigger than a given value, which we arbitrarily posit equal to one, new sellers are likely to add listings to the web-site. We can think they are people who only look at aggregate results before entering a market, non professional vendors drawn from a uniform honesty distribution. If $\Gamma_S < 1$ some of these persons will drop out, with the understanding that $S_h(t)$ be set to zero if it falls below it. The second term $[\Gamma_{S_h}(t) - 1]S_h(t)$ of equation (11) is strongly h-dependent. When it is smaller than one, a percentage $1 - \Gamma_{S_h}$ of sellers of honesty h drops out, vice-versa when $\Gamma_{S_h} > 1$. In this case the newcomers are fairly well informed about the market dynamics and estimate how much extra-profit they can make, thus choosing a specific entry honesty level. Notice that the honesty h_s of a given active seller cannot be changed in time, but s can always exit and come back with a more profitable one.

The functions Γ depend on the probability distribution $\mu(q)$ of the overlap, arising from the choice of the metric space of products, and on the amount of information buyers collect before purchasing an item. In the following sections we shall analyze two particular cases. First we shall model consumers going for one specific product (maximal selection); then flexible ones, looking for a product similar "enough" to their wishes (browsing agents).

3 Maximal selection

Here we analyze a process where potential consumers decide whether to buy or not a single particular item per time unit. As we already mentioned, a buyer b access the web-site looking for a desired product x_b. Now he considers what is available in the market, picks the item that fits best his request, decides whether to buy it or not, and finally he may receive and judge it. Let us assume that, thanks to internal search tools of the web-site, he finds the item $x_{\hat{s}_b}$ corresponding to the maximum overlap $q_{b,\hat{s}_b} = \max_s q_{b,s}$. Then he evaluates it, checking the seller's reputation, and decides if he wants to buy it or not, with no further research. He purchases it with probability $f_b(\hat{s}_b)$, proportional to the buyer's expected reward. The latter can differ from the actual payoff $r_{i,\hat{s}_b}$ (4) he would eventually get from the purchase. In fact, at this stage, the buyer does not have the product $x_{\hat{s}_b}$ in his hands and can only guess upon the available information. He could, therefore, trust differently his perception of h_s and $q_{b,s}$, the first one coming from other buyers' ratings of seller $\hat{s_b}$, the second from a description (sometimes a picture) of the item, provided by the seller himself. Hence we define

$$f_b(s) = h_s^\alpha q_{b,s}, \tag{13}$$

where the exponent α is a parameter that tunes the weight consumers give to sellers' reputation. If he decides to buy, b eventually receives the product, rates seller $\hat{s}_b$ with $h_{\hat{s}_b}$ and is rewarded with a payoff $r_{b,\hat{s}_b}$. The average satisfaction then equals

$$\Gamma_B(t) = \frac{1}{B(t)} \sum_b f_b(\hat{s}_b) r_{b,\hat{s}_b} = \frac{1}{B(t)} \sum_b h_{\hat{s}_b}^{\alpha+1} q_{b,\hat{s}_b}^2. \tag{14}$$

When we take the average over all buyers, we are implicitly averaging over the honesty distribution $p(h,t)$ (9), because the index $\hat{s}_b$ depends on the chosen seller. Let us approximate $q_{b,\hat{s}_b}$ with its average value over all buyers q_{max}; then

$$\Gamma_B(t) = \langle h^{\alpha+1}\rangle q_{max}^2. \tag{15}$$

Every seller has equal probability to maximize the overlap of a given buyer. Conversely their probability to sell a product once chosen, and their unitary profit, depend on their honesty level. The average profit made by a seller of honesty h then reads

$$\begin{aligned}\Gamma_{S_h}(t) &= N_h(t) g_h \\ &= h^\alpha q_{max}\frac{B(t)}{S(t)}(1-h+m),\end{aligned} \tag{16}$$

where $N_h(t) = \sum_{s:h_s=h} n_s(t)/S_h(t)$ is the average number of items sold by a seller of honesty h. According to definition (8), the aggregate satisfaction arising from (16) reads:

$$\Gamma_S(t) = \frac{B(t)}{S(t)}\left[(1+m)\langle h^\alpha\rangle - \langle h^{\alpha+1}\rangle\right]. \tag{17}$$

It is worth noticing the strong feedback effect contained in it: if $S(t)$ becomes much larger than $B(t)$, then $\Gamma_S(t)$ diminishes, thus slowing down the growing rate of $S(t)$ itself. As a consequence a stationary state is reached when $B(t)$ and $S(t)$ grow exponentially with the same exponent, which is entirely determined by $\lim_{t\to\infty}\Gamma_B(t) = \Gamma_B$. An example is given in figure 1.

In the limit of large S we can employ the following approximation:

$$\int_0^{q_{max}} \mu(q)dq \simeq 1 - \frac{1}{S(t)+1}, \tag{18}$$

where $\mu(q)$ is the overlap distribution. Equation (18) becomes exact if $\mu(q)$ is uniform. Let us assume, for the sake of simplicity, that products x_s are real numbers uniformly distributed between zero and one. A natural definition of the distance between two products, on the torus $[0,1]$, is

$$d_{b,s} = \min(|x_b - x_s|, 1 - |x_b - x_s|), \tag{19}$$

which yields the following overlap distribution:

$$\mu(q) = 2\Theta(q - 0.5), \tag{20}$$

where Θ is the Heaviside function. Equation (18) then gives

$$q_{max} = \frac{2S(t) + 1}{2(S(t) + 1)}. \tag{21}$$

We solved numerically equations (10) and (11), with definition (15) for buyers' satisfaction and definition (16), in the approximation (21), for that of sellers. Positing a uniform distribution at time zero, we focused on the honesty distribution of sellers in the stationary regime $p(h) = \lim_{t\to\infty} p(h,t)$. Since $p(h,t)$ (9) results from a natural selection of sellers as a consequence of buyers' behavior, honesties appearing with greater probability reflect higher earnings realized by the corresponding sellers. The lower graph of figure 2 shows a shift of distribution $p(h)$ towards a greater average honesty $\langle h \rangle$, as the value of α is increased. In our model α is the relevant parameter: the larger it is, the more buyers take sellers' reputation into account. In fact the probability $f_b(\hat{s}_b)$ (13) that buyer b actually purchases product $x_{\hat{s}_b}$, decreases for greater α. Such a decrease is not uniform in h, but scales as a power law. As a result, with increasing α sellers with higher honesty are more favored, their relative frequency is enhanced and so is buyer's probability of purchase. The net result of these two competing effects is a greater buyers' satisfaction, in the stationary state, when α is bigger. This appears clearly in figure 3, where the average honesty $\langle h \rangle$ (upper graph) and the buyers' satisfaction Γ_B (lower graph) are shown to be increasing functions of α. As already mentioned, Γ_B determines the slope of both buyers and sellers exponential growth. We conclude that a greater α exerts more selective pressure on sellers, giving rise to a more efficient market and to a faster growth of the web-site usage.

4 Browsing agents

If, instead of considering only the product that maximizes his overlap, a buyer also looks at other offers, he might find better deals. To make things clear, imagine a parameter $\rho \in (0, 0.5]$ tunes the width of customers' search for goods. Among the S items available in the market, buyer b examines the ones ($2S(t)\rho$ on average) closer than ρ to his desired one, i.e. those that fulfill the condition $d_{b,s} < \rho$. This mimics a situation where buyers browse the portion of the web-site containing products they might be interested in. This task is made easy by the division of products into categories, provided by most portals, and by the possibility to display first the ones sold by more reputable sellers. Buyer

b can thus operate a quick selection, after which he picks only one product s, with probability $z_b(s)$, and analyzes it more closely. In the preceding section we analyzed the case $\rho \to 0$, where $z_b(s)$ becomes a Dirac delta function centered in $x_{\hat{s}_b}$. We want to approach here the opposite limit, that of agents performing a wide search before evaluating something for purchase.

Once he has chosen an item $x_{\tilde{s}}$, buyer b proceeds as before: he purchases it with probability $f_b(\tilde{s})$ (13), and is eventually rewarded with $r_{b,\tilde{s}}$ (4). The average buyers' satisfaction over all transactions taking place at time t, namely $\Gamma_B(t)$, then reads

$$\Gamma_B(t) = \frac{1}{B(t)} \sum_b \sum_{s:d_{b,s}<\rho} z_b(s) f_b(s) r_{b,s}. \tag{22}$$

It is sensible to define $z_b(s)$ as a monotonically increasing function of $f_b(s)$. This means the probability of choosing a certain product for evaluation, is proportional to the probability of actually buying it afterwards. This is justified as long as items in the web-site are well organized and sorted. In order to be consistent with such an assumption, the exact functional form of $z_b(s)$ must somehow compensate the density of products available within a given portion of the space. If products are real numbers uniformly distributed in the domain $[0,1]$ and we adopt definition (19) for the distance, then $\mu(q)$ is flat and we can simply set a linear dependence:

$$z_b(s) = \frac{f_b(s)}{\sum_{s:d_{b,s}<\rho} f_b(s)}. \tag{23}$$

Let us define the conditional probability $\mu(q_{b,s}|x_b)$ that a buyer b has overlap $q_{b,s}$ with seller s, given his desire x_b. Inserting equation (23) in (22), and employing definitions (4) and (13), we obtain:

$$\begin{aligned} \Gamma_B(t) &= \frac{1}{B(t)} \sum_b \frac{\sum_{s:d_{b,s}<\rho} h_s^{2\alpha+1} q_{b,s}^3}{\sum_{s:d_{b,s}<\rho} h_s^{\alpha} q_{b,s}} \\ &\to \frac{\langle h^{2\alpha+1}\rangle}{\langle h^{\alpha}\rangle} \int_0^1 dx \frac{\int_0^1 dq \mu(q|x) q^3 \Theta(q-\tilde{\rho})}{\int_0^1 dq \mu(q|x) q \Theta(q-\tilde{\rho})}, \end{aligned} \tag{24}$$

where $\tilde{\rho} = 1-\rho$ and the arrow stands for the limit of large S and B, and for $\rho \gg 1/S$. Similarly we can compute the average profit made by a seller belonging to a certain honesty level h:

$$\Gamma_{S_h}(t) = \frac{g_h}{S_h(t)} \sum_b \frac{\sum_{s:[d_{b,s}<\rho \bigcap h_s=h]} h_s^{2\alpha} q_{b,s}^2}{\sum_{s:d_{b,s}<\rho} h_s^{\alpha} q_{b,s}}$$

$$\to g_h \frac{B(t)}{S(t)} \frac{h^{2\alpha}}{\langle h^\alpha \rangle} \int_0^1 dx \frac{\int_0^1 dq\mu(q|x)q^2\Theta(q-\tilde{\rho})}{\int_0^1 dq\mu(q|x)q\Theta(q-\tilde{\rho})}, \tag{25}$$

where g_h is defined in (2). Here the limit is taken as in (24), with the additional condition $\rho \gg 1/S_h$ for every h.

It is easy to compute the conditional probability $\mu(q_{b,s}|x_b)$. With definition (19) of the distance we obtain, in the continuous limit:

$$\mu(q|x) = \int_0^1 dy \delta(q - \max[|x-y|, 1-|x-y|]) = 2\Theta(q-0.5).$$

Equations (24) and (25) become:

$$\Gamma_B(t) = \frac{1+\tilde{\rho}^2}{2} \frac{\langle h^{2\alpha+1} \rangle}{\langle h^\alpha \rangle} \tag{26}$$

$$\Gamma_{S_h}(t) = \frac{1-\tilde{\rho}^3}{1-\tilde{\rho}^2} \frac{2B(t)}{3S(t)} \frac{h^{2\alpha}}{\langle h^\alpha \rangle} (1-h+m) \tag{27}$$

$$\Gamma_S(t) = \frac{1-\tilde{\rho}^3}{1-\tilde{\rho}^2} \frac{2B(t)}{3S(t)} \frac{1}{\langle h^\alpha \rangle} \left[(1+m)\langle h^{2\alpha} \rangle - \langle h^{2\alpha+1} \rangle \right]. \tag{28}$$

We solved numerically equations (10) and (11) with the above definitions of the Γ-s and with a uniform initial honesty distribution of sellers. In the upper graphs of figures 2 and 4 we show the α and m-dependence of the stable honesty distribution $p(h)$ for browsing agents. The lower graphs of these figures show, as a comparison, simulations with maximal selection. For any given set of the parameters, browsing buyers force sellers to be more honest than q-maximizing ones. This is also shown in the upper graph of figure 3, where the α-dependence of average honesty is displayed. Now we can ask ourselves if also the web-site usage grows more with browsing agents than in the maximal selection case. In the lower graph of figure 3 the stationary buyers' average satisfaction Γ_B, which governs the slope of the exponential growth of $B(t)$ and $S(t)$, is plotted against α. Up to $\alpha \simeq 7.5$ we certainly have a faster growth with browsing agents. A typical snapshot of this situation is given in figure 5, where the stationary honesty distribution and the time growth of $B(t)$ are shown in the two cases. For greater values of α the average honesty approaches a plateau, and so does Γ_B. This limit is rather unrealistic: the overlap q plays nearly no role in the decision of purchase, being dominated by h^α. It becomes, therefore, more profitable to adopt the maximal selection strategy. We should also stress that, in a competitive market, a higher average honesty of sellers would improve the overall web-site reputation, thus increasing the value of c_B and, consequently, the growth rate of $B(t)$. We will, nevertheless, neglect this effect. Finally, figure 6 shows that Γ_B grows with ρ —and so does Γ_S. This confirms the Marriage Problem instance [10]: increased information, even restricted to one side (in our case that of buyers), is beneficial to the whole society.

5 Dynamical equilibrium

It is useful to reformulate the dynamics, i.e. eqs. (10), (11) and (12), in terms of variables

$$\sigma(h,t) = p(h,t)/\Delta h \tag{29}$$
$$\eta(t) = B(t)/S(t), \tag{30}$$

whose time derivatives read:

$$\dot{\sigma}(h,t) = (\Gamma_S(t) - 1) + \sigma(h,t)[\Gamma_{S_h}(t) - 2\Gamma_S(t) + 1] \tag{31}$$
$$\dot{\eta}(t) = \eta(t)[\Gamma_B(t) - 2\Gamma_S(t) + 1 + c_B]. \tag{32}$$

These variables eventually reach a constant value, due to the equilibration of two sets of competing effects. First, that of sellers' honesty: a greater h level enhances the probability of selling a product (see (13)), but reduces the unitary gain $g_h = 1 + m - h$ (2). Second, that of the ratio buyers/sellers: a bigger $\eta(t)$ means there are more buyers for each seller. This increases the average sellers' satisfaction $\Gamma_S(t)$, which in turn makes $S(t)$ increase, and $\eta(t)$ diminish. The stationarity condition yields:

$$\Gamma_B + c_B - 1 = 2(\Gamma_S - 1) \tag{33}$$
$$\sigma(h) = \frac{(\Gamma_S - 1)}{2\Gamma_S - \Gamma_{S_h} - 1}, \tag{34}$$

from which it is clear that the inequality $\Gamma_B > 1 - c_B$ must hold to ensure growth. Equation (34) is the result of our darwinian-type selection, which implies that the most frequent h-population be the most fit (satisfied). From equation (16) (resp. (27)) we can compute the mode h_m^{MS} (resp. h_m^{BA}) of distribution $p(h)$:

$$h_m^{MS} = \frac{(1+m)\alpha}{1+\alpha} \tag{35}$$
$$h_m^{BA} = \frac{2(1+m)\alpha}{1+2\alpha}. \tag{36}$$

When the mode equals one, fully honest sellers have an advantage over the others. If that happens for a given set of parameters (m, α), agents' satisfaction approaches a limit value. In figure 3 this is shown, in particular, for the α-dependence of Γ_B. Equations (35) and (36) explain why the plateau value is reached faster with browsing agents.

In order to find the stationary honesty distribution, we should solve equation (33) for η and substitute the result into (34). For the case of browsing agents, by inserting expressions (26), (27) and (28) in equations (33) and (34), we obtain:

$$\eta = \frac{(c_B+1)\langle h^\alpha\rangle + a_1\langle h^{2\alpha+1}\rangle}{2a_2\langle u_\alpha(h)\rangle} \tag{37}$$

$$\sigma(h) = \left\{1 + \left[1 - \frac{u_\alpha(h)}{\langle u_\alpha(h)\rangle}\right] v_\alpha\right\}^{-1}, \tag{38}$$

where $a_1 = \frac{1+\tilde{\rho}^2}{2}$, $a_2 = \frac{2(1-\tilde{\rho}^3)}{3(1-\tilde{\rho}^2)}$ and

$$u_\alpha(h) = h^{2\alpha}(1+m) - h^{2\alpha+1}$$

$$v_\alpha = \left[1 - \frac{\langle h^\alpha\rangle}{a_2\eta\langle u_\alpha(h)\rangle}\right]^{-1} = 1 + \frac{2\langle h^\alpha\rangle}{a_1\langle h^{2\alpha+1}\rangle + (c_B-1)\langle h^\alpha\rangle}.$$

Now equation (38) can be solved self-consistently.

Similarly, for the case of maximal selection, we insert equations (15), (16) and (17) into (33), we eliminate η and substitute the expressions thus obtained in (34). Finally we end up with the following equation:

$$\left[(\langle h^{\alpha+1}\rangle + c_B + 1)\left(1 - \frac{\tilde{u}_\alpha(h)}{2\langle\tilde{u}_\alpha(h)\rangle}\right) - 1\right]\sigma(h) = \frac{\langle h^{\alpha+1}\rangle + c_B - 1}{2}, \tag{39}$$

where $\tilde{u}_\alpha(h)$ is given by

$$\tilde{u}_\alpha(h) = h^\alpha(1+m) - h^{\alpha+1}. \tag{40}$$

The above relation (39) can be also solved self-consistently. An example is given in figure 7, where the theoretical stationary distribution arising from (39) is shown to match exactly the one found solving numerically the original time dependent differential equations, (10) and (11), with the same set of parameters. All other stationary quantities can be calculated accordingly.

6 Honesty vs Reputation

In the preceding sections we assumed reputation equals honesty. The two could, in fact, differ for the following main reasons. The first source of problem relies in imprecise consumers' ratings, but it is the minor one if the volume of affairs is big, since mistakes have no preferential direction. Moreover Δh can be chosen of the same order of magnitude as the variance of individual mistakes, thus identifying h with the consumers' average judgment. Second comes cheating, that is a seller, who has so far been good, might occasionally sell at an higher price. This could temporarily improve the gain of some seller, but it should only affect the variance and not the average satisfaction of buyers in the stationary regime. Thirdly, the reputation building process could be very inaccurate. We shall concentrate on the latter because it seems to be the main shortcoming of some commercial web-sites existing today.

Let us consider the extreme case, although common, where the rating form available in the web-site allows buyers to state if they made a good bargain or not, with no further specification. As a result, reputation consists in being good (h_g) or bad (h_b), and this is the only information about sellers buyers are provided with. Once they purchased a product, though, buyers can evaluate it accurately and judge it according to their proper honesty scale. Therefore the "true" honesty level h still plays the same role here as in equation (4), whereas elsewhere it must be substituted by $\tilde{h}$, the two levels reputation. Equations (26), (27) and (28) then become:

$$\Gamma_B(t) = \frac{1+\tilde{\rho}^2}{2}\frac{\langle h\tilde{h}^{2\alpha}\rangle}{\langle \tilde{h}^{\alpha}\rangle} \tag{41}$$

$$\Gamma_{S_h}(t) = \frac{1-\tilde{\rho}^3}{1-\tilde{\rho}^2}\frac{2B(t)}{3S(t)}\frac{\tilde{h}^{2\alpha}}{\langle \tilde{h}^{\alpha}\rangle}(1-h+m) \tag{42}$$

$$\Gamma_S(t) = \frac{1-\tilde{\rho}^3}{1-\tilde{\rho}^2}\frac{2B(t)}{3S(t)\langle \tilde{h}^{\alpha}\rangle}\left[(1+m)\langle \tilde{h}^{2\alpha}\rangle - \langle h\tilde{h}^{2\alpha}\rangle\right], \tag{43}$$

where $\tilde{h} = h_g$ if $h \geq 1/2$ and $\tilde{h} = h_b$ if $h < 1/2$.

In this situation sellers less honest than 0.5 tend to die out. For the higher intrinsic honesty levels, those who are closer to 0.5 are favored, and $p(h)$ decays exponentially toward $h = 1$. This defect of information transmission, something like a narrow channel effect [11], damages severely the web-site usage. In fig. 8 we plotted the time evolution of $B(t)$ in this binary case, with $h_g = 1 - \Delta h$ and $h_b = \Delta h$, together with the case of browsing agents with perfect judging forms at their disposal. It is clear that the latter case shows a much faster growth.

7 Comments

We have shown, within our model, that a good rating form can help the growth of a commercial web-site, overcoming the problem of asymmetrical information. But, how is buyers' browsing ability influenced by its architecture? A good categorization of products is, of course, important: this way we would probably approach the most profitable region of figure 3. We believe a major step forward would be achieved once it will be possible to guess accurately buyers' future wishes [12,13].

The equations we studied, i.e. (10) and (11), can be regarded as mean-field approximations to a stochastic behavior. We suppose, on average, an exponential growth of the web-site usage: this might mimic a fast growing stage of e-commerce web-sites. We believe the role of honesty and information we tried to stylize here applies to any situation where a great number of sellers is easily reachable to any buyer.

Our calculations are carried out by identifying buyers and sellers with real numbers: this is a useful simplification, but it is easy to substitute them with bit strings. In this case the distance (19) becomes the hemming distance, and probability (23) should be redefined appropriately.

Full information and unlimited processing capability of buyers could, in principle, allow them to maximize directly the product hq. Let us imagine each buyer b follows the maximal selection strategy, with $\hat{s}_b = [\hat{s} : h_{\hat{s}} q_{b,\hat{s}} = \max_s h_s q_{b,s}]$. This would favor so much honest sellers that the honesty distribution $p(h)$ would become a delta function centered in $h = 1$, which corresponds to a perfectly efficient market.

8 Acknowledgements

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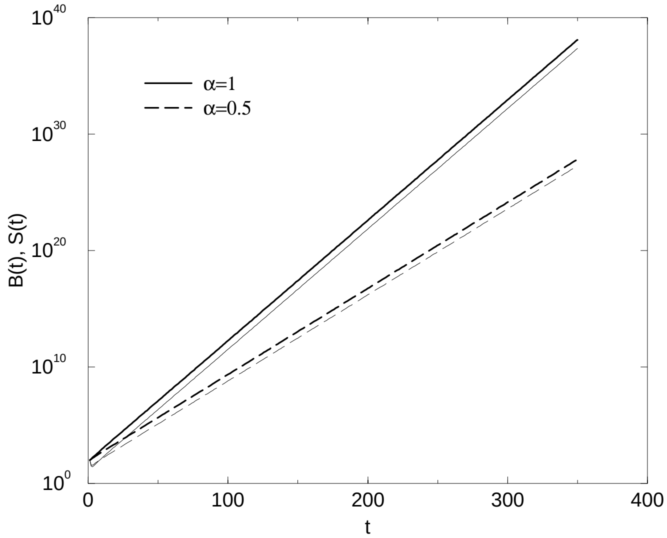

Fig. 1. Buyers (bold lines) and sellers growth as a function of time, with maximal selection.

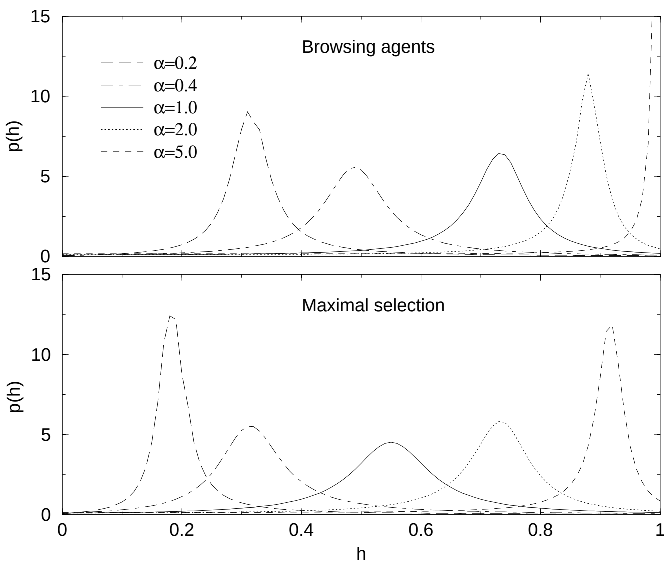

Fig. 2. Stationary honesty distribution of sellers with maximal selection (lower graph) and browsing agents with $\rho = 0.5$ (upper graph). Different line-styles correspond to different values of α: the legend refers to both graphs. We fixed $H = 100$, $m = 0.1$ and $c_B = 0.9$. Normalization of $p(h)$ is set to 100.

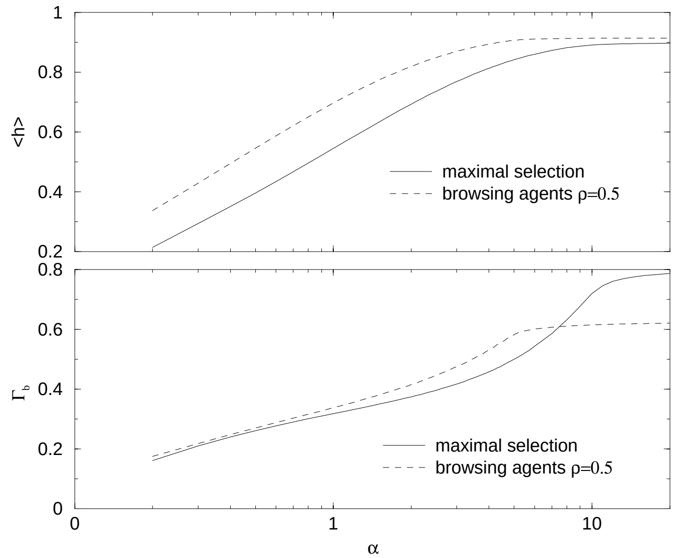

Fig. 3. Upper graph: average honesty of sellers, in the stationary state, as a function of α. Lower graph: average buyers' satisfaction Γ_B, in the stationary state, as a function of α. We fixed $H = 100$, $m = 0.1$ and $c_B = 0.9$. The logarithmic x-axis scale is the same for both graphs.

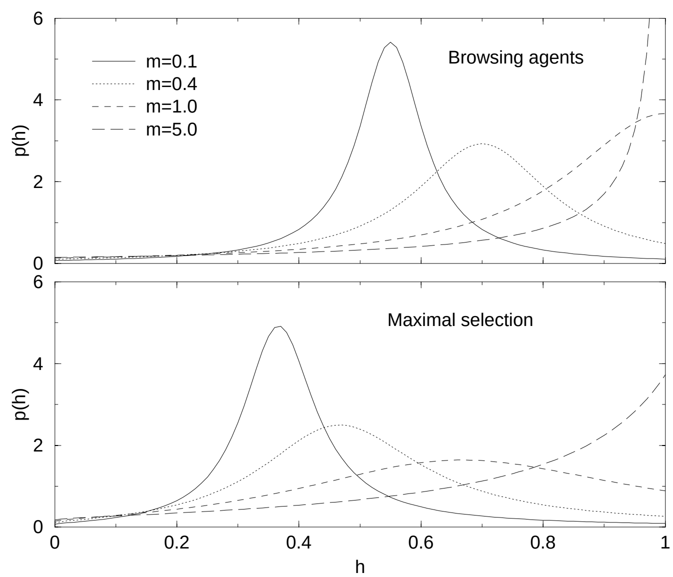

Fig. 4. Stationary honesty distribution of sellers with maximal selection (lower graph) and browsing agents with $\rho = 0.5$ (upper graph). Different line-styles correspond to different values of the profit margin m: the legend refers to both graphs. We fixed $H = 100$, $\alpha = 0.5$ and $c_B = 0.9$. Normalization of $p(h)$ is set to 100.

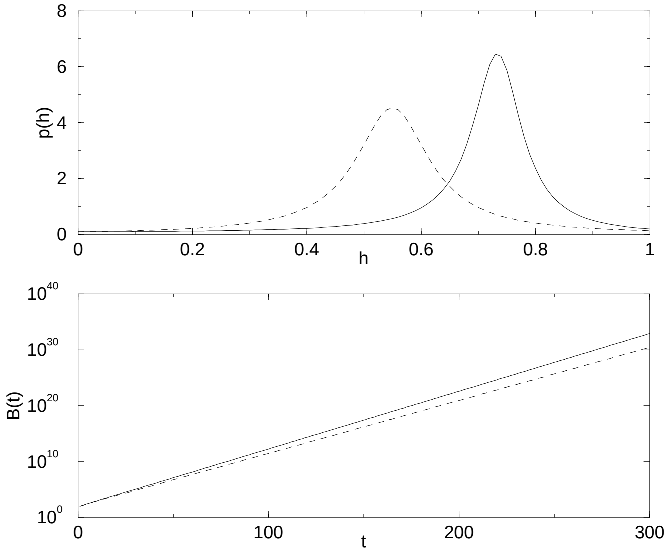

Fig. 5. Buyers' time evolution (lower graph) and stable honesty distribution of sellers (upper graph). Solid lines are browsing angents simulations with $\rho = 0.5$, while dashed ones are with maximal selection. In both cases we fixed $H = 100$, $m = 0.1$ and $\alpha = 1$.

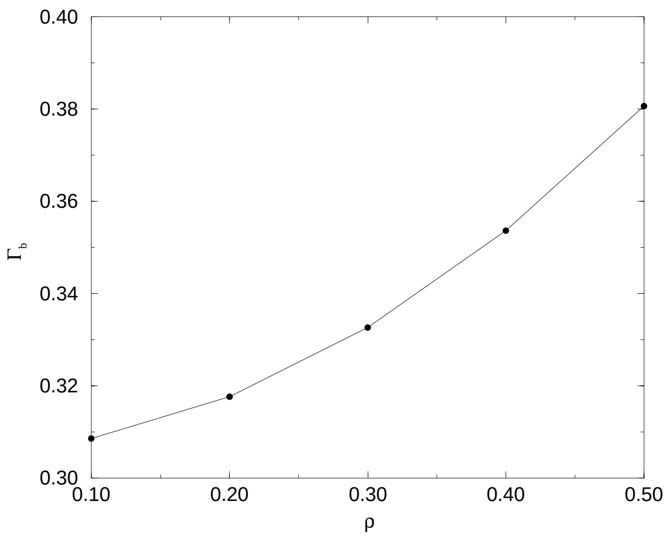

Fig. 6. Average buyers' satisfaction Γ_B for browsing agents, in the stationary state, as a function of ρ. We fixed $H = 100$, $m = 0.1$, $c_B = 0.9$ and $\alpha = 1.5$.

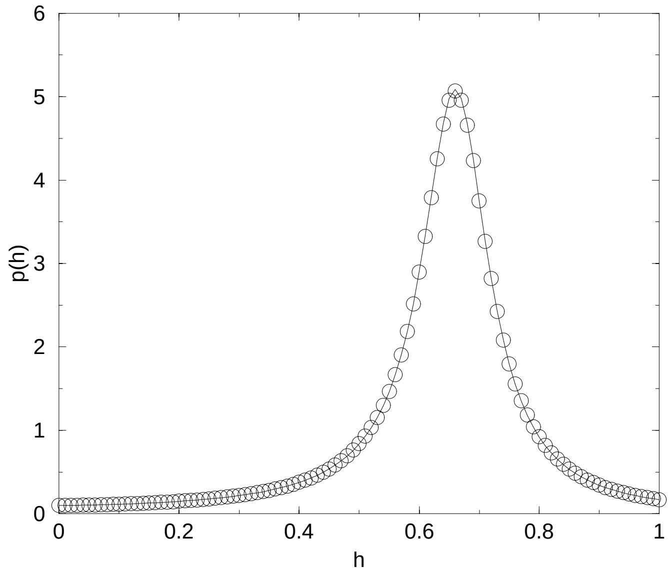

Fig. 7. Honesty distribution of sellers with maximal selection. Circles are numerical simulations, the solid line comes from equation (39). The parameters are: $H = 100$, $c_B = 0.1$, $\alpha = 1.5$ and $m = 0.1$.

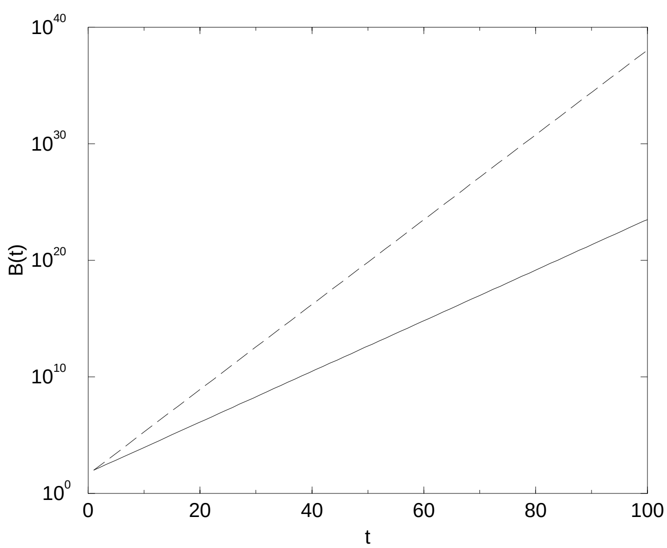

Fig. 8. Buyers growth as a function of time for browsing agents. The dashed line represent the case $\tilde{h} = h$, the solid line the binary case $\tilde{h} = h_g, h_b$. The parameters are: $H = 100$, $c_B = 1.5$, $\alpha = 1$ and $m = 0.1$, in both cases.